Orbital Evolution of an Accreting Millisecond Pulsar: Witnessing the Banquet of a Hidden Black Widow?

T. Di Salvo[1*], L. Burderi[2⋆], A. Riggio[2], A. Papitto[3,4], M.T. Menna[3]

[1] *Dipartimento di Scienze Fisiche ed Astronomiche, Università degli Studi di Palermo, via Archirafi 36 - 90123 Palermo, Italy*

[2] *Dipartimento di Fisica, Università degli Studi di Cagliari, SP Monserrato-Sestu, KM 0.7, 09042 Monserrato, Italy*

[3] *I.N.A.F. - Osservatorio Astronomico di Roma, via Frascati 33, 00040 Monteporzio Catone (Roma), Italy*

[4] *Dipartimento di Fisica, Università degli Studi di Roma 'Tor Vergata', via della Ricerca Scientifica 1, 00133 Roma, Italy*

ABSTRACT

We have performed a timing analysis of all the four X-ray outbursts from the accreting millisecond pulsar SAX J1808.4−3658 observed so far by the PCA on board RXTE. For each of the outbursts we derived the best-fit value of the time of ascending node passage. We find that these times follow a parabolic trend, which gives an orbital period derivative $\dot{P}_{\rm orb} = (3.40 \pm 0.18) \times 10^{-12}$ s/s, and a refined estimate of the orbital period, $P_{\rm orb} = 7249.156499 \pm 1.8 \times 10^{-5}$ s (reference epoch $T_0 = 50914.8099$ MJD). This derivative is positive, suggesting a degenerate or fully convective companion star, but is more than one order of magnitude higher than what is expected from secular evolution driven by angular momentum losses caused by gravitational radiation under the hypothesis of conservative mass transfer. Using simple considerations on the angular momentum of the system, we propose an explanation of this puzzling result assuming that during X-ray quiescence the source is ejecting matter (and angular momentum) from the inner Lagrangian point. We have also verified that this behavior is in agreement with a possible secular evolution of the system under the hypothesis of highly non-conservative mass transfer. In this case, we find stringent constraints on the masses of the two components of the binary system and its inclination. The proposed orbital evolution indicates that in this kind of sources the neutron star is capable to efficiently ablate the companion star, suggesting that this kind of objects are part of the population of the so-called black widow pulsars, still visible in X-rays during transient mass accretion episodes.

1   INTRODUCTION

SAX J1808.4−3658 is the first discovered among the ten known accreting millisecond pulsars (hereafter AMSPs), which are all transient X-ray sources, and is still the richest laboratory for timing studies of this class of objects. Although timing analysis have been now performed on most of the sources of this sample with interesting results (see Di Salvo et al. 2007 for a review and references therein), SAX J1808.4−3658 is the only known AMSP for which several outbursts have been observed by the RXTE/PCA with high time resolution. In particular, the first outburst of this source was observed by the RXTE/PCA in April 1998, when coherent X-ray pulsations at ~ 2.5 ms and orbital period of ~ 2 hr (Wijnands & van der Klis 1998; Chakrabarty & Morgan 1998) were discovered. The source showed other X-ray outbursts in 2000 (when only the final part of the outburst could be observed, Wijnands et al. 2001), in 2002 (when kHz QPOs and quasi-coherent oscillations during type-I X-ray bursts were discovered, Wijnands et al. 2003; Chakrabarty et al. 2003), and again in 2005, approximately every two years (see Wijnands 2005 for a review).

Although widely observed, SAX J1808.4−3658 remains one of the most enigmatic sources among AMSPs, since timing analyses performed on this source have given puzzling results. Burderi et al. (2006), analysing the 2002 outburst, found that the pulse phases (namely the pulse arrival times) show evident shifts, probably caused by variations of the pulse profile shape. They noted that the phases derived from the second harmonic of the pulse profile were much more stable, and tentatively derived a spin frequency derivative from these data. To explain the relatively large frequency derivative a quite high mass accretion rate was required, about a factor of 2 higher than the extrapolated bolometric luminosity of the source during the same outburst. Hartman et al. (2008, hereafter H08) performed a timing analysis of all the four outbursts of SAX J1808.4−3658 observed up to date by RXTE, finding again complex phase shifts in all of them. Their conclusion was that the large variations of the pulse shape do not allow to infer any spin frequency evolution during a single outburst, with typical upper limits of $|\dot{\nu}| \lesssim 2.5 \times 10^{-14}$ Hz/s (95% c.l.), which were derived excluding the first few days of the 2002 and 2005 outbursts and the large residuals at

---

⋆ E-mail:disalvo@fisica.unipa.it, burderi@mporzio.astro.it

the 2002 mid-outburst. Interestingly, combining the results from all the analysed outbursts, H08 found a secular spin frequency derivative of $\dot{\nu} = (-5.6 \pm 2.0) \times 10^{-16}$ Hz/s, indicating a secular spin-down of the neutron star in this system. From this measure they found an upper limit of 1.5×10^8 Gauss to the neutron star magnetic field.

Papitto et al. (2005) performed a temporal analysis of the outbursts of SAX J1808.4$-$3658 that occurred in 1998, 2000, and 2002, which resulted in improved orbital parameters of the system. The large uncertainty caused by the relatively limited temporal baseline made it impossible to derive an estimate of the orbital period derivative. In this paper we use all the four outbursts of SAX J1808.4$-$3658 observed by RXTE/PCA, spanning a temporal baseline of more than 7 years, to derive an orbital period derivative, the first reported to date for an AMSP. The value we find with high statistical significance is surprising, $\dot{P}_{\mathrm{orb}} = (3.40 \pm 0.18) \times 10^{-12}$ s/s. This value for the orbital period derivative is compatible with the measure reported by H08, which, independently and simultaneously, have found the orbital period derivative of SAX J1808.4$-$3658. In § 3 we propose a simple explanation of this result arising from considerations on the conservation of the angular momentum of the system, which is consistent with a (non-conservative) secular evolution of the system. In particular, we hypothesize that during quiescence SAX J1808.4$-$3658 experiences a highly non-conservative mass transfer, in which a great quantity of mass is lost from the system with a relatively high specific angular momentum.

2   TIMING ANALYSIS AND RESULTS

In this paper we analyse RXTE public archive data of SAX J1808.4$-$3658 taken during the April 1998 (Obs. ID P30411), the February 2000 (Obs. ID P40035), the October 2002 (Obs. ID P70080), and the June 2005 (Obs. ID P91056 and Obs. ID P91418) outbursts, respectively (see Wijnands 2005 and H08 for a detailed description of these observations). In particular, we analysed data from the PCA (Jahoda et al. 1996), which is composed of a set of five xenon proportional counters operating in the $2-60$ keV energy range with a total effective area of 6000 cm^2. For the timing analysis, we used event mode data with 64 energy channels and a 122 μs temporal resolution. We considered only the events in the $3-13$ keV energy range where the signal to noise ratio is the highest, but we checked that a different choice (considering for instance the whole RXTE/PCA energy range) does not change the results described below. The arrival times of all the events were referred

to the solar system barycenter by using JPL DE-405 ephemerides along with spacecraft ephemerides. This task was performed with the faxbary tool, considering as the best estimate for the source coordinates the radio counterpart position, that has a 90% confidence error circle of 0.4 arcsec radius, which is compatible with that of the optical counterpart (Rupen et al. 2002; Giles et al. 1999).

For each of the outbursts we derived a precise orbital solution using standard techniques (see e.g. Burderi et al. 2007; Papitto et al. 2007, and references therein). In particular, we firstly corrected the arrival times of all the events with the orbital solution given by Papitto et al. (2005). Then we looked for differential corrections to the adopted orbital parameters as described in the following. We epoch-folded time intervals with a duration of about 720 s each (1/10 of the orbital period) at the spin period of 2.49391975936 ms, and fitted each pulse profile obtained in this way with sinusoids in order to derive the pulse arrival times or pulse phases. The folding period was kept constant for all the outbursts. In fact, the determination of the pulse phases is insensitive to the exact value of the period chosen to fold the light curves providing that this value is not very far from the true spin period of the pulsar. Note that H08 have measured a secular derivative of the spin frequency in SAX J1808.4−3658, that is $\dot{\nu} = (-5.6 \pm 2.0) \times 10^{-16}$ Hz/s. This is a quite small value, and they do not find evidence of variations of the spin frequency during a single outburst. In order to choose a value of the spin period as close as possible to the true one, we used the value above, that is in between the best-fit spin period reported by Chakrabarty & Morgan (1998) for the 1998 outburst and the best-fit spin period reported by Burderi et al. (2006) for the 2002 outburst. As in Burderi et al. (2006), we fitted each pulse profile with two sinusoids of fixed periods (the first, with period fixed to the spin period adopted for the folding, corresponding to the fundamental, and the second, with period fixed to half the spin period, corresponding to the first overtone, respectively). In all cases the $\chi^2/d.o.f.$ obtained from the fits of the pulse profiles was very close to (most of the times less than) 1. In order to improve the orbital solution we used the phase delays of the fundamental of the pulse profile which has the best statistics; the uncertainties on these phases were derived calculating 1-σ statistical errors from the fit with two sinusoids.

We then looked for differential corrections to the adopted orbital parameters, which can be done by fitting the pulse phases as a function of time for each outburst. In general, any residual orbital modulation is superposed to a long-term variation of the phases, e.g. caused by a variation of the spin. However, as noted by Burderi et al. (2006) for the 2002 outburst,

SAX J1808.4−3658 shows a very complex behavior of the pulse phases with time, with phase shifts, probably caused by variations of the pulse shape, that are difficult to model and to interpret. To avoid any fitting of this complex long-term variation of the phases, we preferred to restrict the fit of the differential corrections to the orbital parameters to intervals in which the long-term variation and/or shifts of phases are negligible. We therefore considered consecutive intervals with a duration of at least 4 orbital periods (depending on the statistics), and fitted the phases of each of these intervals with the formula for the differential corrections to the orbital parameters (see e.g. Deeter, Pravdo & Boynton 1981 and eq. (3) in Papitto et al. 2007). The selection of the intervals is somewhat arbitrary; we have verified, however, that the results do not change using a different choice. No significant corrections were found on the adopted values of the orbital period, P_{orb}, the projected semimajor axis of the neutron star (NS) orbit, $a_1 \sin i/c$, and the eccentricity of the orbit. In particular, for the eccentricity we find an upper limit of 4.6×10^{-5} (95% c.l.) combining all the data.

On the other hand we found that the times of passage of the NS at the ascending node at the beginning of each outburst, T_N^*, were significantly different from their predicted values, $T_0^* + N P_{\mathrm{orb}}$ where T_0^* is the adopted time of ascending node passage at the beginning of the 1998 outburst and the integer N is the exact number of orbital cycles elapsed between two different ascending node passages, i.e. N is the integer part of $(T_N^* - T_0^*)/P_{orb}$ under the assumption that $|T_N^* - (T_0^* + N P_{\mathrm{orb}})| < P_{orb}$ that we have also verified *a posteriori*. We therefore fixed the values of P_{orb}, $a_1 \sin i/c$, and the eccentricity, and derived the differential corrections, ΔT^*, to the time of passage at the ascending node, obtaining a cluster of points for each outburst, which are plotted as a function of time in the inset of Figure 1. This has been done in order to check that (systematic) uncertainties on the arrival times of the pulses (such as phase shifts or other kind of noise) not already included in our estimated uncertainties for the pulse phases, did not significantly affect the determination of the orbital parameters. Since each of the points corresponding to an outburst may be considered like an independent estimate of the same quantity, if the errors on the phases were underestimated, we would expect that errors on the derived orbital parameters were underestimated. In this case the scattering of the points representing the time of passage at the ascending node derived for each of the considered intervals would be larger than the errors associated with each point. This indeed is not the case; in the inset of Fig. 1, we have shown the results obtained from each of these intervals, and the scattering of the points appears always
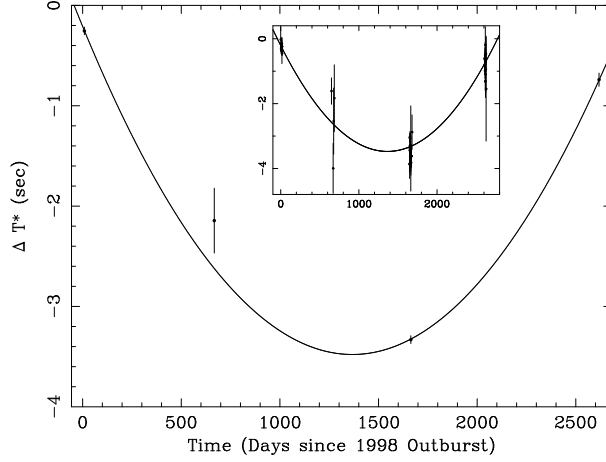
Figure 1. Differential correction, ΔT^*, to the time of passage of the NS at the ascending node for each of the four outbursts analysed. In the inset we show the single measurements of ΔT^* obtained for each of the consecutive time intervals in which each outburst has been divided (see text). All the times are computed with respect to the beginning of the 1998 outburst, $T_0 = 50914.8099$ MJD.

comparable to the associated 1-σ error. The largest scattering is observed for the outburst of 2000, which indeed is the point with the largest distance from the best fit parabola (see below). We think that this is caused by the worse statistics during this observation, which was taken at the end of the 2000 outburst.

We hence combined all the measurements corresponding to each outburst computing the error-weighted mean of the corresponding points, obtaining the four points shown in Figure 1. These points show a clear parabolic trend that we fitted to the formula:

$$\Delta T^* = \delta T_0^* + \delta P_{orb} \times N + (1/2)\dot{P}_{orb}P_{orb} \times N^2 \tag{1}$$

In this way we found the best fit values $T_0^* + \delta T_0^*$, $P_{\rm orb} + \delta P_{\rm orb}$ and $\dot{P}_{\rm orb}$ at $t = T_0$ shown in Table 1, with a $\chi^2 = 2.2$ (for 1 d.o.f.). This corresponds to a probability of 13% of obtaining a χ^2 larger than the one we found. Our result is therefore acceptable (or, better, not rejectable), since the probability we obtained is above the conventionally accepted significance level of 5% (in this case, in fact, the discrepancy between the expected and observed values of χ^2 is not significant since the two values are within less than 2 σ from each other, see e.g. Bevington & Robinson 2003). We find a highly significant derivative of the orbital period, which indicates that the orbital period in this system is increasing at a rate of $(3.40 \pm 0.12) \times 10^{-12}$ s/s. Note that H08, simultaneously and independently, found a very similar result for the orbital period derivative in SAX J1808.4$-$3658, $\dot{P}_{orb} = (3.5 \pm 0.2) \times 10^{-12}$ s/s, compatible with the result reported in this paper. The only difference is in the quoted error. H08 state that

Table 1. Best fit orbital parameters for SAX J1808.4−3658.

Parameter	Value
T_0^* (days)	50914.8784320(11)
P_{orb} (s)	7249.156499(18)
$\dot{P}_{orb}$ (s/s)	$3.40(18) \times 10^{-12}$
$a_1 \sin i$ (lt-ms)	$62.809(1)^a$
e	$< 4.6 \times 10^{-5}$
χ^2/dof	0.98/1

The reference time at which the orbital period, P_{orb} and its derivative, $\dot{P}_{\mathrm{orb}}$, are referred to is the beginning of the 1998 outburst, that is $T_0 = 50914.8099$ MJD. Numbers in parentheses are the uncertainties in the last significant digits at 90% c.l. Upper limits are at 95% c.l. Uncertainties are calculated conservatively increasing the errors on the fitted points in order to reach a $\chi^2/dof \simeq 1$, as described in the text.
[a] The value of $a_1 \sin i$ and its 1-σ error are from Chakrabarty & Morgan 1998.

this difference is possibly due to an underestimate of the phase uncertainties reported in this paper, as testified by a worse reduced χ^2. We just note that we do not have any evidence that the phase uncertainties we derive are underestimated. Indeed our reduced χ^2 is worse than the one obtained by H08, that is $\chi^2 = 1.01$ for 1 dof), but it is still statistically acceptable. However, in the hypothesis that our errors are slightly underestimated, we have increased by a factor of 1.5 all the errors on the phases in order to obtain a χ^2 as close as possible to 1. In this way we obtain a χ^2 of 0.98 for 1 d.o.f., and we have re-evaluated the errors on the orbital parameters, finding that these errors increase by a factor of 1.5. This means that our "conservative" estimate of the orbital period derivative is $(3.40 \pm 0.18) \times 10^{-12}$ s/s (90% c.l.). As a final check we have fitted with the same formula the points shown in the inset of Figure 1, obtaining, as expected, the same results.

Note that the orbital period of SAX J1808.4−3658 is now known with a precision of 1 over 10^9, that is an improvement of one order of magnitude respect to the previous estimate by Papitto et al. (2005) and two orders of magnitude with respect to Chakrabarty & Morgan (1998). On the other hand, the derivative of the orbital period indicates that the orbital period in this system is increasing at the quite large rate of $(3.40 \pm 0.18) \times 10^{-12}$ s/s, that is at least an order of magnitude higher than what is predicted by a conservative mass transfer driven by Gravitational Radiation (GR, see below). In the next section we discuss a possible explanation for this surprising result.

3   DISCUSSION

We have performed a precise timing analysis of all the X-ray outbursts of the AMSP SAX J1808.4−3658 observed to date by the RXTE/PCA, covering more than 7 years in time. We divided each outburst in several intervals and found, for each interval, differential corrections to previously published orbital parameters. The obtained times of passage of the NS at the ascending node were significantly different in different outbursts. We fitted these times with a parabolic function of time finding an improved orbital solution valid over a time span of more than 7 years. This solution includes a highly significant derivative of the orbital period, $\dot{P}_{orb} = (3.40 \pm 0.18) \times 10^{-12}$ s/s. This value, found simultaneously and independently by H08, is the first measure of the orbital period derivative for an AMSP. However, this orbital period derivative is quite unexpected, since it is more than one order of magnitude higher than what is expected from conservative mass transfer driven by GR.

Orbital evolution calculations show that the orbital period derivative caused by conservative mass transfer induced by emission of GR is given by:

$$\dot{P}_{\rm orb} = -1.4 \times 10^{-13} \, m_1 \, m_{2,\,0.1} \, m^{-1/3} P_{2h}^{-5/3} \left[(n - 1/3)/(n + 5/3 - 2q) \right] \, ss^{-1} \tag{2}$$

(derived from Verbunt 1993; see also Rappaport et al. 1987), where m_1 and m are, respectively, the mass of the primary, M_1, and the total mass, $M_1 + M_2$, in units of solar masses, $m_{2,\,0.1}$ is the mass of the secondary in units of 0.1 $M_\odot$, P_{2h} is the orbital period in units of 2 h, $q = m_2/m_1$ is the mass ratio and where n is the index of the mass-radius relation of the secondary $R_2 \propto M_2^{\rm n}$. Therefore a positive orbital period derivative certainly indicates a mass-radius index $n < 1/3$, and therefore, most probably, a degenerate or fully convective companion star (see e.g. King 1988). However, the $\dot{P}_{\rm orb}$ we measure is an order of magnitude higher than what is expected from GR.

3.1   Conservation of angular momentum

In order to explain the quite unexpected value measured for the orbital period derivative, we start from the equation of the angular momentum of the system, which must be verified instantaneously. The orbital angular momentum of the system can be written as: $J_{\rm orb} = [Ga/(M_1 + M_2)]^{1/2} M_1 M_2$, where G is the Gravitational constant, and a is the orbital separation. We can differentiate this expression in order to find the variation of the orbital angular momentum of the system caused by mass transfer. We indicate with $-\dot{M}_2$ the mass transferred by the secondary, which can be accreted onto the neutron star (conservative mass

transfer) or can be lost from the system (non-conservative mass transfer). We can therefore write $\dot{M_1} = -\beta \dot{M_2}$, where β is the fraction of the transferred mass that is accreting onto the neutron star, while $1 - \beta$ is the fraction of the transferred mass that is lost from the system. The specific angular momentum, l_{ej}, with which the transferred mass is lost from the system can be written in units of the specific angular momentum of the secondary, that is: $\alpha = l_{ej}/(\Omega_{orb} r_2^2) = l_{ej} P_{orb}(M_1 + M_2)^2/(2\pi a^2 M_1^2)$, where r_2 is the distance of the secondary star from the center of mass of the system. Calculating the derivative of the orbital angular momentum of the system, we obtain:

$$\frac{\dot{P}_{orb}}{P_{orb}} = 3 \left[\frac{\dot{J}}{J_{orb}} - \frac{\dot{M_2}}{M_2} g(\beta, q, \alpha) \right], \tag{3}$$

where $g(\beta, q, \alpha) = 1 - \beta q - (1 - \beta)(\alpha + q/3)/(1 + q)$, and $\dot{J}/J_{orb}$ represents possible losses of angular momentum from the system (e.g. caused by GR). Since the term $\dot{J}/J_{orb}$ must be negative, while the measured $\dot{P}_{orb}/P_{orb}$ is positive, we can derive a lower limit on the positive quantity $-\dot{M_2}/M_2$ assuming that $\dot{J}/J_{orb} = 0$:

$$\frac{\dot{P}_{orb}}{P_{orb}} \leqslant 3 \left(-\frac{\dot{M_2}}{M_2} g(\beta, q, \alpha) \right). \tag{4}$$

Assuming a conservative mass transfer, $\beta = 1$, it is easy to see that $g(1, q, \alpha) = 1 - q \simeq 1$, where we have used the information that for SAX J1808.4−3658 the mass function gives $q \geqslant 4 \times 10^{-2}$ for $M_1 = 1.4~M_\odot$ (Chakrabarty & Morgan 1998), and can be therefore neglected. Hence, for conservative mass transfer the conservation of angular momentum gives: $\dot{P}_{orb}/P_{orb} \leqslant 3(-\dot{M_2}/M_2)$. We can estimate the averaged mass transfer rate from the averaged observed luminosity of the source. Since SAX J1808.4−3658 accretes for about 30 days every two years, we have estimated an order of magnitude for the averaged X-ray luminosity from the source, that is $L_X \sim 4 \times 10^{34}$ ergs/s, which corresponds to $3(-\dot{M_2}/M_2) = 6.6 \times 10^{-18}~\text{s}^{-1}$. It is easy to see that the measured $\dot{P}_{orb}/P_{orb}$, $4.7 \times 10^{-16}~\text{s}^{-1}$, is at least 70 times higher than the value predicted in the conservative mass transfer case, hence excluding this scenario.

Assuming a totally non-conservative mass transfer, $\beta = 0$, we find that $g(0, q, \alpha) = (1 - \alpha + 2q/3)/(1 + q) \simeq 1 - \alpha$, implying that $\dot{P}_{orb}/P_{orb} \leqslant 3(1 - \alpha)(-\dot{M_2}/M_2)$. Since the first term is positive we find that $\alpha < 1$ and the specific angular momentum with which matter is expelled from the system must be less than the specific angular momentum of the secondary. For matter leaving the system with the specific angular momentum of the primary we have $\alpha = q^2 \sim 0$. In this case we find, therefore, the same result of the conservative case where

no angular momentum losses from the system occur. This is due to the fact that the specific angular momentum of the primary is so small that there is no difference with respect to the conservative case. Since the specific angular momentum of the mass lost must be in between the specific angular momentum of the primary and that of the secondary, a reasonable hypothesis is that matter leaves the system with the specific angular momentum of the inner Lagrangian point. In this case $\alpha = [1 - 0.462(1+q)^{2/3}q^{1/3}]^2 \simeq 0.7$, where we have used for the Roche Lobe radius the approximation given by Paczyński (1971). We therefore find $\dot{P}_{orb}/P_{orb} \leqslant (-\dot{M}_2/M_2)$. Using the measured value of the quantity $\dot{P}_{orb}/P_{orb} = 4.7 \times 10^{-16}$ s^{-1}, we find that to explain this result in a totally non-conservative scenario the mass transfer rate must be: $(-\dot{M}_2) = \dot{M}_{ej} \geqslant 8.3 \times 10^{-10}\ M_\odot\ yr^{-1}$. We can therefore explain the measured derivative of the orbital period of the system assuming that the system is expelling matter at a quite large rate, that may be as high as $\sim 10^{-9}\ M_\odot\ yr^{-1}$, and this is found just assuming the conservation of the angular momentum of the system, and independently of the secular evolution adopted.

3.2 Possible secular evolution of the system

In order to verify whether this result is just a transient peculiar behavior of the system due to unknown causes or it is instead compatible with a secular evolution, we have solved the secular evolution equations of the system using the assumptions described in the following. i) Angular momentum losses are due to GR and are given by: $\dot{J}/J = -32G^3M_1M_2(M_1 + M_2)/(5c^5a^4)$, where c is the speed of light, and $J = M_1M_2[Ga/(M_1 + M_2)]^{1/2}$ is the binary angular momentum (see e.g. Landau & Lifschitz 1958; Verbunt 1993). ii) For the secondary we have adopted a mass-radius relation $R_2 \propto M_2^n$. iii) We have imposed that the radius of the secondary follows the evolution of the secondary Roche Lobe radius: $\dot{R}_{L2}/R_{L2} = \dot{R}_2/R_2$, where for the radius of the secondary Roche Lobe we have adopted the Paczyński (1971) approximation: $R_{L2} = 2/3^{4/3}[q/(1+q)]^{1/3}a$ that is valid for small mass ratios, $q = M_2/M_1 \leqslant 0.8$. In these hypotheses we derived a simple expression for the orbital period derivative and the mass transfer rate in the extreme cases of totally conservative and totally non-conservative mass transfer (see e.g. Verbunt 1993; van Teeseling & King 1998; King et al. 2003; King et al. 2005):

$$\dot{P}_{orb} = -1.38 \times 10^{-12} \left[\frac{n - 1/3}{n - 1/3 + 2g}\right] m_1^{5/3} q(1+q)^{-1/3} P_{2h}^{-5/3}\ \ \text{s/s} \tag{5}$$

$$\dot{M} = -\dot{M}_2 = 4.03 \times 10^{-9} \left[\frac{1}{n - 1/3 + 2g} \right] m_1^{8/3} q^2 (1+q)^{-1/3} P_{2h}^{-8/3} \ \ M_\odot/\text{yr} \tag{6}$$

where $g = 1 - q$ for totally conservative mass transfer (in this case it is easy to see that eq. 5 gives the same $\dot{P}_{orb}$ given by eq. 2, as expected), and $g = 1 - (\alpha + q/3)/(1 + q)$ for totally non-conservative mass transfer.

Comparing the measured orbital period derivative with eq. 5 (assuming that the orbital period derivative we measure reflects the secular evolution of the system), we note that, in order to have $\dot{P}_{orb} > 0$ we have to assume an index $n < 1/3$. In the case of SAX J1808.4$-$3658 the secondary mass is $m_2 \leqslant 0.14$ at 95% c.l. (Chakrabarty & Morgan 1998), and therefore the mass ratio $q \leqslant 0.1$ implies that for the totally conservative mass transfer case ($g = 1 - q$), the $\dot{P}_{orb}$ expected from GR must be of the order of 10^{-13} s/s, not compatible with what we measured for SAX J1808.4$-$3658. In other words, if we assume a conservative mass transfer for the system (that means that the mass transferred during outbursts is completely accreted by the NS, and during quiescence no or negligible mass is accreted or lost from the system), we find that it is impossible to explain the observed orbital period derivative with a secular evolution driven by GR.

On the other hand, if we assume that during X-ray quiescence the companion star is still overflowing its Roche Lobe but the transferred mass is not accreted onto the NS and is instead ejected from the system, we find a good agreement between the measured and expected orbital period derivative assuming that the matter leaves the system with the specific angular momentum at the inner Lagrangian point, $\alpha = [1 - (2/3^{4/3})q^{1/3}(1+q)^{2/3}]^2$. Adopting the measured value $\dot{P}_{orb} = 3.4 \times 10^{-12}$ s/s and the other parameters of SAX J1808.4$-$3658, eq. 5 translates into a relation between m_1, m_2 and the mass-radius index n; this is plotted in Figure 2 (top panel) for different values of n going from 0 to $n = -1/3$. The constraint on m_1 vs. m_2 imposed by the mass function of the system is also plotted (the shadowed region in the figure) and indicates that the most probable value for n is $-1/3$, which in turn indicates a degenerate or, most probably, a fully convective companion star. In fact, in a system with orbital period less than 3 h, where the mass of the Roche-Lobe filling companion is below $0.2 - 0.3$ $M_\odot$, the companion star becomes fully convective with a mass-radius hydrostatic equilibrium equation $R \propto M^{-1/3}$ (e.g. King 1988; Verbunt 1993). Also, for reasonable minimum, average and maximum values of the NS mass, 1.1, 1.56, and 2.2 $M_\odot$, respectively, we obtain the following values for the secondary mass: 0.053, 0.088, and 0.137 $M_\odot$, and the following values for the inclination of the system: 44°, 32°, and 26°, respectively.

Assuming therefore $n = -1/3$ we have plotted in Figure 2 (bottom panel) the corresponding non-conservative mass transfer rate as a function of m_1. We find that for $m_1 = 1.5$ the mass transfer rate must be of the order of 10^{-9} $M_\odot$/yr, much higher than what is expected in a conservative GR driven mass transfer case. Note that this high $\dot{M}$ might explain the spin period evolution reported by Burderi et al. (2006; see, however, H08 who could not detect a spin period derivative during the outbursts). Actually, during the X-ray outbursts, the mass transfer is conservative since the transferred matter is accreted onto the NS. However, the accretion phase duty cycle, about 40 days / 2 years = 5%, is so small that the totally non-conservative scenario proposed above is a good approximation.

3.3   Is SAX J1808.4−3658 a 'hidden' black widow pulsar?

If the hypothesis of a highly non-conservative mass transfer in SAX J1808.4−3658 is correct, the question to answer is why is accretion inhibited during X-ray quiescence while the companion star is transferring mass at a high rate? We propose that the answer has to be found in the radiation pressure of the magneto-dipole rotator emission, with a mechanism that is similar to what is proposed to explain the behavior of the so-called black widow pulsars (see e.g. Tavani et al. 1991a; King et al. 2003; 2005; see also Burderi et al. 2001). Indeed, the possibility that the magneto-dipole emission is active in SAX J1808.4−3658 during X-ray quiescence has been invoked by Burderi et al. (2003, see similar results in Campana et al. 2004) to explain the optical counterpart of the source, which is observed to be over-luminous during quiescence (Homer et al. 2001). In this scenario, the optical luminosity in quiescence is explained by the spin-down luminosity of the magneto-dipole rotator (with a magnetic field of $(1-5) \times 10^8$ Gauss) which is reprocessed by the companion star and/or a remnant accretion disc. Interestingly, similar evidence of a strongly irradiated companion star during quiescence has been found also for IGR J00291+5934, the fastest among the known AMSPs (D'Avanzo et al. 2007).

In other words, a temporary reduction of the mass-accretion rate onto the neutron star (note that the so-called disc Instability Model, DIM – see e.g. Dubus, Hameury, & Lasota 2001 – usually invoked to explain the transient behavior of these sources, may play a role in triggering or quenching the X-ray outbursts in SAX J1808.4−3658) may cause the switch on of the emission of the magneto-dipole rotator, and, in some cases, even if the mass transfer rate has not changed, the accretion of matter onto the NS can be inhibited because the
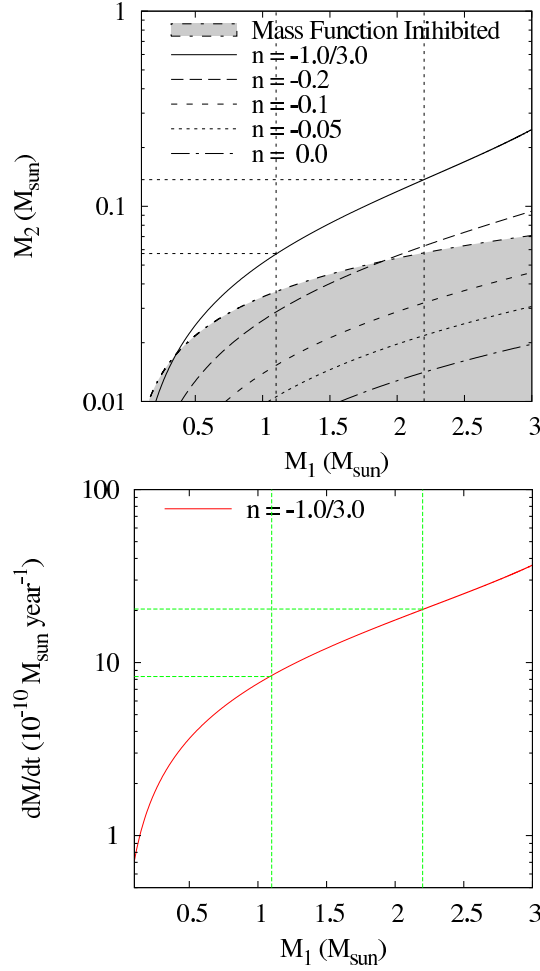
Figure 2. Top: Companion star mass vs. NS mass in the hypothesis of totally non conservative mass transfer (with matter leaving the system with the specific angular momentum at the inner Lagrangian point) and assuming the $\dot{P}_{orb}$ measured for SAX J1808.4−3658. Different curves correspond to different values of the mass-radius index n of the secondary. Horizontal lines indicate the limits for the secondary star mass corresponding to reasonable limits for the NS mass and to $n = -1/3$. **Bottom:** Mass rate outflowing the secondary Roche Lobe in the hypothesis of totally non conservative mass transfer (as above) and assuming $n = -1/3$.

radiation pressure from the radio pulsar may be capable of ejecting out of the system most of the matter overflowing from the companion (see e.g. Burderi et al. 2001 and references therein). This phase has been termed "radio–ejection". One of the strongest predictions of this model is the presence, during the radio-ejection phase, of a strong wind of matter emanating from the system: the mass overflowing from the companion swept away by the radiation pressure of the pulsar. Indeed, the existence of 'hidden' millisecond pulsars, whose radio emission is completely blocked by material engulfing the system that is continuously replenished by the mass outflow driven by companion irradiation, has already been predicted by Tavani (1991a).

A beautiful confirmation of this model was provided by the discovery of PSR J1740–5340, an eclipsing millisecond radio pulsar, with a spin period of 3.65 ms, located in the globular cluster NGC 6397 (D'Amico et al. 2001). It has the longest orbital period ($P_{\rm orb} \simeq 32.5$ hrs) among the 10 eclipsing pulsars detected up to now. The peculiarity of this source is that the companion is a slightly evolved turnoff star still overflowing its Roche lobe. This is demonstrated by the presence of matter around the system that causes long lasting and sometimes irregular radio eclipses, and by the shape of the optical light curve, which is well modeled assuming a Roche-lobe deformation of the mass losing component (Ferraro et al. 2001). An evolutionary scenario for this system has been proposed by Burderi, D'Antona, & Burgay (2002), who provided convincing evidence that PSR J1740–5340 is an example of a system in the radio-ejection phase, by modeling the evolution of the possible binary system progenitor. In other words, PSR J1740–5340 can be considered a 'star-vaporizing pulsar' of type II in the terminology used by Tavani (1991a).

We believe that the behavior of SAX J1808.4−3658 is very similar to the one of PSR J1740–5340, the main differences being the orbital period, which is ~ 32 h in the case of PSR J1740–5340 and ~ 2 h in the case of SAX J1808.4−3658, and the mass transfer rate from the companion, which has been estimated to be $\sim 10^{-10}$ $M_\odot$ yr^{-1} for PSR J1740–5340 and one order of magnitude higher for SAX J1808.4−3658. Both these factors will increase the local Dispersion Measure (DM) to the source in the case of SAX J1808.4−3658, and hence will predict a much higher free-free absorption in the case of SAX J1808.4−3658. This is in agreement with the fact that, although widely searched, no radio pulsations have been detected from SAX J1808.4−3658 up to date (Burgay et al. 2003). In other words, SAX J1808.4−3658 can be considered a 'hidden' millisecond pulsar, or a 'star-vaporizing pulsar' of type III in the terminology used by Tavani (1991a).

A similar highly non conservative mass transfer, triggered by irradiation of the secondary and/or of an accretion disc by the primary (according to the model of Tavani et al. 1991b), has been proposed to explain the large orbital period derivative observed in the ultra-compact Low Mass X-ray Binary X 1916–053, composed of a neutron star and a semi-degenerated white dwarf, exhibiting periodic X-ray dips. In this case, $\dot{P}_{orb}/P_{orb} \simeq 5.1 \times 10^{-15}$ s^{-1}, which implies a mass transfer rate of $\sim 10^{-8}$ $M_\odot$ yr^{-1}, with $60 - 90\%$ of the companion mass loss outflowing from the system (Hu, Chou, & Chung 2008).

As predicted by several authors (e.g. Chakrabarty & Morgan 1998; King et al. 2005), and in agreement with our interpretation of the orbital period derivative in SAX J1808.4−3658 as

due to a highly non-conservative mass transfer, we propose therefore that SAX J1808.4−3658, and other similar systems, belong to the population of the so-called black widow pulsars (or are evolving towards black widow pulsars); these are millisecond radio pulsars thought to ablate the companion and likely able to produce large mass outflows. When (or if) the pressure of the outflowing matter becomes sufficiently high to temporarily overcome the radiation pressure of the magneto-dipole rotator, the source experiences a transient mass accretion episode, resulting in an X-ray outburst. Indeed, SAX J1808.4−3658 and the other known AMSPs are all transient systems (accreting just for a very short fraction of the time), with small values of the mass function (implying small minimum mass for the secondary) and short orbital periods (less than a few hours).[1]

Although in some of these black widow radio pulsars (variable) radio eclipses have been observed, clearly demonstrating the presence of matter around the system, a direct proof of severe mass losses from these system has never been found to date. The orbital evolution of SAX J1808.4−3658 indicates that this X-ray transient millisecond pulsar indeed may expel mass from the system for most of the time with just short episodes of accretion observed as X-ray outbursts. We therefore propose that SAX J1808.4−3658 (and perhaps most AMSPs) is indeed a black widow still eating the companion star.

However, some black widow pulsars have shown quite complex derivatives of the orbital period. In particular, the prototype of this class, PSR B1957+20 (a type-I star-vaporizing eclipsing millisecond pulsar), shows a large first derivative of the orbital period (almost an order of magnitude higher than the one of SAX J1808.4−3658), and also a second orbital period derivative, indicating a quasi-cyclic orbital period variation (Arzoumanian, Fruchter, & Taylor 1994). A similar behavior has been observed in another binary millisecond pulsar, PSR J2051–0827, which shows a third derivative of the orbital period and a variation of $a \sin i$, indicating that the companion is underfilling its Roche lobe by $\sim 50\%$ (Doroshenko et al. 2001). In these cases, the complex orbital period variation has been ascribed to gravitational quadrupole coupling (i.e. a variable quadrupole moment of the companion which is due to a cyclic spin-up and spin-down of the star rotation). In this scenario, the companion star must be partially non-degenerate, convective and magnetically active, so that the wind

---

[1] Among the presently known AMSPs, the newly discovered intermittent pulsar SAX J1748.9–2021 (Altamirano et al. 2008) in the globular cluster NGC 6440 is an exception since this pulsar shows mass function and orbital period higher than the other Galactic AMSP. However, it is worth noting that this pulsar belongs to a globular cluster in which capture in the dense cluster environment may have played a role.

of the companion star will result in a strong torque which tends to slow down the star, making the companion star rotation deviating from synchronous rotation ($\Omega_C = f2\pi/P_{\rm orb}$, with $f < 1$).

In the case of SAX J1808.4−3658, the present data do not allow to find a second derivative of the orbital period, and therefore it is not clear if the orbital period derivative will change sign. However, we note that for a (type-I) black widow (radio) pulsar the companion star may be no more strongly orbitally locked; hence deviations from co-rotation may be possible, and this may reflect in strange (cyclic) changes of the orbital period of the system. However, in the case of SAX J1808.4−3658, that is an accreting neutron star, the companion must be completely orbitally locked (since the secondary star still fills its Roche lobe during X-ray outbursts and cannot detach in a time scale of a few years) and therefore the most probable way to change the orbital period of the system is a change of the averaged specific angular momentum, which can be obtained transferring mass from the secondary to the neutron star or expelling mass from the system with appropriate specific angular momentum, less than the specific angular momentum of the secondary.

In conclusion, we propose that we are witnessing the behavior of a 'hidden' black widow, eating its companion during X-ray outbursts and ablating it during quiescence; the next X-ray outburst of SAX J1808.4−3658 will be of fundamental importance to test or disprove this scenario. Moreover, the analogy of SAX J1808.4−3658 with a black widow should also be tested observationally; observing the source at other wavelength may give important information. For instance, it is already known that SAX J1808.4−3658 shows transient radio emissions; this was observed for the first time at the end of the 1998 outburst, approximately 1 day after the onset of a rapid decline in the X-ray flux, by Gaensler, Stappers, & Getts (1999) and was attributed to an ejection of material from the system. The possible presence of an $H\alpha$ bow shock nebula (like the one observed in PSR B1957+20) may be difficult to test in this case, given the crowding around the source in the optical band (see Campana et al. 2004), while it may be important to look for an IR excess which may be caused by excess of matter around the system.

REFERENCES

Altamirano D., Casella P., Patruno A., Wijnands R., van der Klis M. 2008, ApJ, 674, L45

Arzoumanian Z., Fruchter A. S., Taylor J. H. 1994, ApJ, 426, L85

Bevington P.R., & Robinson D. K., 2003, Data Reduction and Error Analysis for the Physical Sciences, 3rd Ed., Boston, MA: McGraw-Hill, ISBN

Burderi L., Possenti A., D'Antona F., Di Salvo T., Burgay M., Stella L., Menna M. T., Iaria R., Campana S., D'Amico N. 2001, ApJ, 560, L71

Burderi L., D'Antona F., Burgay M. 2002, ApJ, 574, 325

Burderi L., Di Salvo T., D'Antona F., Robba N. R., Testa V. 2003, A&A, 404, L43

Burderi L., Di Salvo T., Menna M. T., Riggio A., Papitto A. 2006, ApJ, 653, L133

Burderi L., Di Salvo T., Lavagetto G., Menna M. T., Papitto A., Riggio A., Iaria R., D'Antona F., Robba N. R., Stella L. 2007, ApJ, 657, 961

Burgay M., Burderi L., Possenti A., D'Amico N., Manchester R. N., Lyne A. G., Camilo F., Campana S. 2003, ApJ, 589, 902

Campana S., D'Avanzo P., Casares J., et al. 2004, ApJ, 614, L49

Chakrabarty D., & Morgan E.H., 1998, Nature, 394, 346

Chakrabarty D., Morgan E. H., Muno M. P., Galloway D. K., Wijnands R., van der Klis M., Markwardt C. B. 2003, Nature, 424, 42

D'Amico N., Possenti A., Manchester R. N., Sarkissian J., Lyne A. G., Camilo F. 2001, ApJ, 561, L89

D'Avanzo P., Campana S., Covino S., Israel G. L., Stella L., Andreuzzi G. 2007, A&A, 472, 881

Deeter J. E., Boynton P. E., Pravdo S. H. 1981, ApJ, 247, 1003

Di Salvo T., Burderi L., Riggio A., Papitto A., Menna M. T. 2007, AIP Conference Proceedings, in press (arXiv:0705.0464)

Doroshenko O., Lohmer O., Kramer M., Jessner A., Wielebinski R., Lyne A. G., Lange Ch. 2001, A&A, 379, 579

Dubus G., Hameury J.-M., Lasota J.-P. 2001, A&A, 373, 251

Ferraro F. R., Possenti A., D'Amico N., Sabbi E. 2001, ApJ, 561, L93

Gaensler B. M., Stappers B. W., & Getts T. J. 1999, ApJ, 522, L117

Giles A.B., Hill K.M., Greenhill J.G., 1999, MNRAS, 304, 47

Hartman, J. M., Patruno A., Chakrabarty D., et al. 2008, ApJ, 675, 1468 (H08)

Homer L., Charles P. A., Chakrabarty D., van Zyl L., 2001, MNRAS, 325, 1471

Hu C.-P., Chou Y., Chung Y.-Y. 2008, ApJ, in press (arXiv:0712.1868)

Jahoda K., et al., 1996, in *Proc. SPIE*, 2808, 59

King, A. R., 1988, QJRAS, 29, 1

King A. R., Davies M. B., Beer M. E. 2003, MNRAS, 345, 678

King A. R., Beer M. E., Rolfe D. J., Schenker K., Skipp J. M. 2005, MNRAS, 358, 1501

Landau L. D., & Lifshitz E. M. 1975, in The classical theory of fields

Paczynski, B. 1971, ARA&A, 9, 183

Papitto A., Menna M.T., Burderi L., Di Salvo T., D'Antona F., Robba N.R., 2005, ApJ, 621, L113

Papitto A., Di Salvo T., Burderi L., Menna M. T., Lavagetto G., Riggio A. 2007, MNRAS, 375, 971

Rappaport S., Ma C. P., Joss P. C., Nelson L. A. 1987, ApJ, 322, 842

Rupen M.P., Dhawan V., Mioduszewski A.J., Stappers B.W., Gaensler B.M., 2002, IAUC 7997, 2

Tavani M. 1991a, ApJ, 379, L69

Tavani M. 1991b, Nature, 351, 39

van Teeseling A., King A. R. 1998, A&A, 338, 957

Verbunt, F. 1993, ARA&A, 31, 93

Wijnands R., & van der Klis M., 1998, Nature, 394, 344

Wijnands R., Mendez M., Markwardt C., van der Klis M., Chakrabarty D., Morgan E. 2001, ApJ, 560, 892

Wijnands R., van der Klis M., Homan J., Chakrabarty D., Markwardt C. B., Morgan E. H. 2003, Nature, 424, 44

Wijnands R., 2005, in *Pulsars New Research*, Nova Science Publishers (NY)